Math 55 Discussion problems 11 Apr

1. How many different messages can be transmitted in n microseconds using three different signals if one signal requires 1 microsecond for transmittal, the other two signals require 2 microseconds each for transmittal, and a signal in a message is followed immediately by the next signal?
2. Find the solution to $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n = 3, 4, 5, ...$, with $a_0 = 3$, $a_1 = 6$, and $a_2 = 0$.
3. (a) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$.
   (b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 4$.
4. Suppose that there are two goats on an island initially. The number of goats on the island doubles every year by natural reproduction, and some goats are either added or removed each year.
   (a) Construct a recurrence relation for the number of goats on the island at the start of the n^{th} year, assuming that during each year an extra 100 goats are put on the island.
   (b) Solve the recurrence relation from part (a) to find the number of goats on the island at the start of the n^{th} year.
   (c) Construct a recurrence relation for the number of goats on the island at the start of the n^{th} year, assuming that n goats are removed during the n^{th} year for each $n \geq 3$.
   (d) Solve the recurrence relation in part (c) for the number of goats on the island at the start of the n^{th} year.